CNPP Fuel Rod Vibration Analysis Using Finite Element Method

Shakir Hussain[1], M. Rafique[2], Anwar Ahmad[3] and Syed Waseem Akhtar[4]

Abstract

The fuel rods in the PWR fuel assembly are continuously supported by spacer grids, which are located at intervals along the axial length of the fuel assembly. The coolant flow induces vibration in fuel rod which can wear away its cladding tube surface in contact with the support structure. Thus, it is important to examine the characteristics of the fuel rod vibrations. In the present study, two different methodologies have been developed to calculate the natural frequencies and mode shapes of the 300 MWe CNPP fuel rod. In the first methodology, a 2D planner flexure modal analysis of the fuel rod is performed using MATLAB software, while in the second, a 3D modal analysis is carried out using ANSYS software. In 3D analysis, actual geometry of the CNPP fuel rod and spacer grid spring/dimple support is modeled. The vibration analysis of pre-stressed model is carried out in the modal analysis module of ANSYS. The purpose of this study is not only to compare the 2D and 3D FE models but also to verify the results with the vibration measurements made by the Chinese for CNPP fuel rod. Both the 2D MATLAB and 3D ANSYS FE analyses results are found to be in good agreement with the Chinese test results and thus validate the analyses models.

Keywords: Fuel Rod, Spacer Grid, FE, Model Analysis, Natural Frequencies, Mode Shapes

Introduction

In a Pressurized Water Reactor (PWR), the fuel rods are supported at intervals along their lengths by spacer grids which maintain the lateral spacing between the rods. Each fuel rod is supported by six contact support points provided in each spacer grid (SG) cell by the combination of dimples and springs. The fuel rods are exposed to stringent operating conditions such as high temperature, high pressure and the massive coolant flow velocity (3~8m/s) during its designed lifetime (typically 3 cycles). The structural integrity of fuel rod cladding is very important being the first barrier to the release of radioactive fission products into the atmosphere.

The vibration of fuel rod is mainly generated by the high coolant flow which produces energy to induce vibrations. Since the coolant normally flows axially upward and parallel to the fuel rods, therefore, the vibration behavior of fuel rods is called axial flow-induced vibration (FIV). It is known that the vibration source in the reactor exists in wide range of 0 to 50 Hz Kenard M. W et al, (1995) which can make resonance causing potential damage to the fuel rod. Thus, it is important to understand the fuel rod vibration behavior in the reactor.

In order to prevent fuel failure due to the external excitation, the natural frequencies of the fuel rod are evaluated to assure that these are not matched with the expected excitation frequencies. These mechanisms must be prevented by adequate design of the fuel assembly structure (i.e., using an appropriate number of grid supports per fuel assembly and providing adequate stiffness) based on the coolant flow conditions in the reactor core. The principle design concern is not the prevention of the vibration mechanism but the limitation of resultant wear at the grid-to-rod support interface, which is produced due to relative motion (or sliding) between fuel rod cladding and SG.

The phenomena of fuel rod vibration have analyzed using FE analysis by various researchers. M.H Choi et al (2004) described the vibration analysis of continuously supported dummy fuel rod. H. S. Kang et al, Preumont. A, (1980) performed modal analysis for the newly designed SG. Comprehensive vibration behavior of PWR rods is studied by Andre Preumont Kang H. S. et al, (2001). Experimental study on vibration of flexible cylinders induced by axial flow is studied by M. P. Paidoussis Paidoussis M. P., (1968). However, it is still difficult to get the exact solutions for the fuel rod vibration supported by springs and dimples.

In the present study, a finite element (FE) model of the fuel rod has been developed to analyze vibration behavior of Chashma Nuclear Power Plant (CNPP) fuel rod. Two different FE models are adopted to calculate the modal parameters of the fuel rod. In first method FE based computer program

1-3 Directorate General Nuclear Power Fuel, Pakistan Atomic Energy Commission

is developed in MATLAB to calculate the eigenvalues and eigenvectors for the single fuel rod supported with eight SGs at intervals along its axial length. In this model, the fuel rod is replaced with 2D beam element and the SG dimples and springs are replaced with elastic spring elements. In the second method, a 3D modal analysis of the CNPP fuel rod is performed using ANSYS 13.0 software. The FE model of single fuel rod supported with eight SGs is generated as per actual geometry. Same material properties and boundary conditions are applied as in the previous case. For model validation, the results of both the analyses are also compared with the Chinese test results for CNPP fuel rod.

Description of Fuel Assembly

The CNPP fuel assembly consists of a 15×15 square array having 204 fuel rods, 20 guide thimbles, 1 instrumentation tube, 8 SGs, a bottom nozzle and a top nozzle. The fuel rod consists of slightly enriched uranium dioxide ceramic pellets contained in zircaloy-4 tubing which is plugged and seal welded at the ends to encapsulate the fuel. The guide thimbles and instrumentation tube in conjunction with 8 SGs and nozzles form the structural part, i.e. Skeleton, of the fuel assembly. The fuel rods are loaded into the skeleton such that there is clearance between the fuel rod ends and the top and bottom nozzles to accommodate irradiation induced growth and thermal expansion.

In the fuel assembly skeleton, the fuel rods are supported by SG springs and dimples. The main function of the SGs is not only to support the fuel rods at specific locations or intervals along their length but also to maintain the lateral spacing between the rods (rod to rod distance) throughout the designed life of the fuel assembly. In each SG cell, a fuel rod is supported by six contact support points (two springs and four dimples). The plane view of the fuel rod along with the isometric views of the SG contact arrangements with the fuel rod are shown in Figures 1 and 2, respectively.

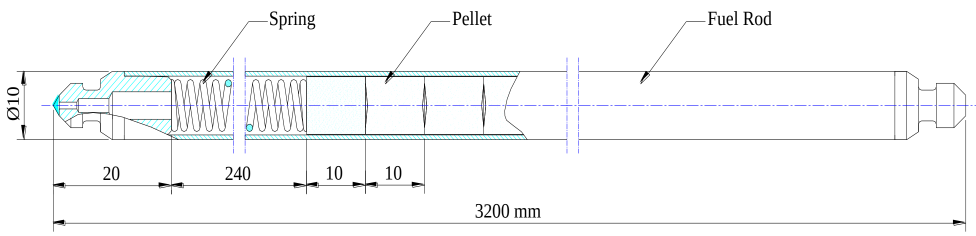

Figure 1: CNPP Fuel Rod

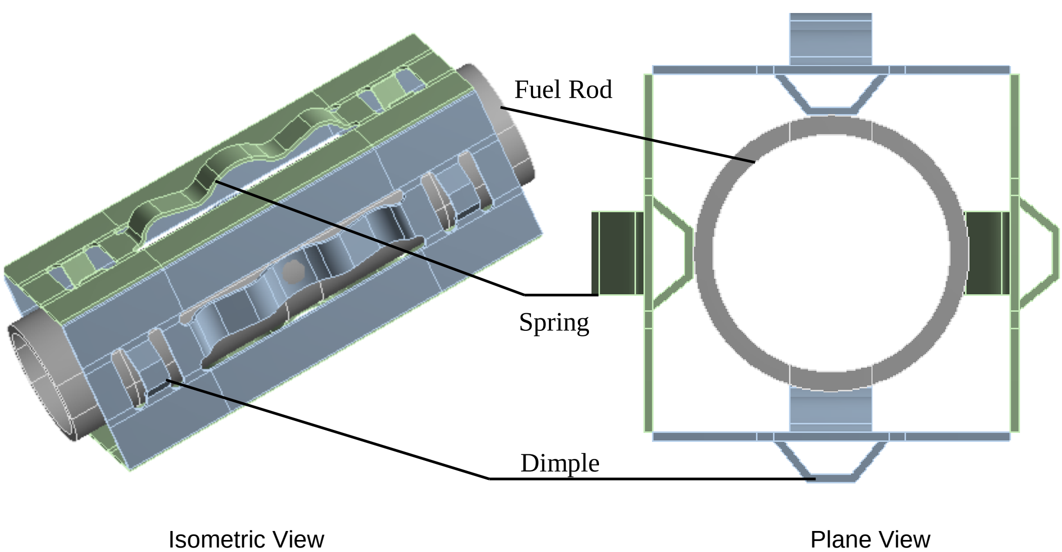

Figure 2: CNPP Spacer Grid Cell Showing the Contact Arrangements

FE Model and Analyses

In order to simplify the 2D & 3D models for determining the natural frequencies & modes shapes of the CNPP fuel rod, following assumptions are made:

i) During vibration, the fuel rod is always in contact with the springs and dimples of SG cell.
ii) The fuel stack does not contribute in the bending moment Kang H. S. et al, (2001).
iii) The mass per unit length includes both the cladding and fuel.

2-D Modal Analysis using MATLAB

In order to perform the 2D modal analysis using MATLAB software, it is assumed that the fuel rod exhibits only planner flexure vibration and its vibration plane is consistent with the plane determined by its supports structures, i.e. the springs and dimples in the SG cell are located in perpendicular planes to the fuel rod. The fuel rod is considered as 2D continuous beam element and the SG springs and dimples are replaced with 2D elastic spring elements with linear stiffness. Since, the CNPP fuel rod is supported by eight SGs, therefore, it can be regarded as seven span continuous beam Choi M. H. et al, (2004). The fuel rod is discretized in such a way that three nodes are taken at each SG cell (two at dimple and one at spring). Five nodes are taken in each span between two adjacent SGs; therefore, as a result 59 nodes are taken altogether in this continuous beam. The FE model of the CNPP fuel rod supported in a SG cell is shown in Figure 3.

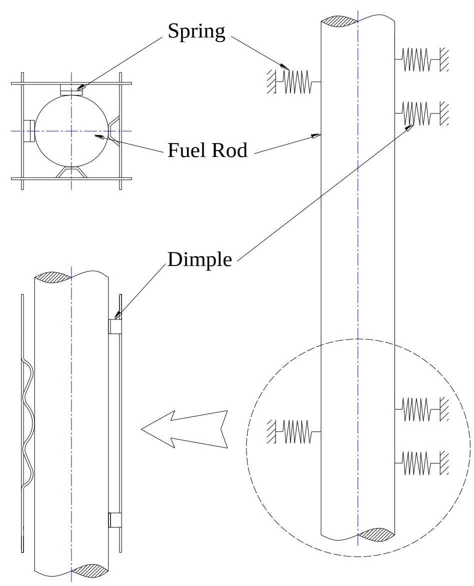

Figure 3: FE Model of CNPP Fuel Rod

Mathematical Model

The equation of the fuel rod vibration without damping is

$$[M]\{\ddot{X}\} + [K]\{X\} = 0 \qquad \text{Eq. (1)}$$

Where $\{X\}=\{V_1,V_2,V_3,.......,V_n,\ \ \Phi_1,\Phi_2,\Phi_3,....\Phi_n\}^T$, $[M]$ is global mass matrix and $[K]$ is global stiffness matrix. Assuming displacement function;

$$\{X\} = \{U\}\{sin(wt + \Phi)\}$$

Simplifying Eq. (1) the terms

$$\{U\} sin(wt+\Phi)([K]- w^2[M])=0$$

$$[K]\{U\}-w^2[M]\{U\}=0 \qquad \text{Eq. (2)}$$

For the purpose of distinguishing the non-null mass term from the null mass term, vector $\{U\}$ is divided into two parts, i.e. $\{U\}=\{U_1, \ U_2\}^T$. Where, $\{U_1\}=\{\bar{V}_1, \ \bar{V}_2, \ .... \ \bar{V}_n\}^T$ includes 'n' DoF (Degree of freedom) of non-null mass terms; and $\{U_2\}=\{\bar{\Phi}_1, \ \bar{\Phi}_2, \ .. \ \bar{\Phi}_n\}^T$ includes 'n' DoF of null mass terms. Correspondently, the stiffness matrix and mass matrix are divided into four parts respectively, i.e.

$$K=EI\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \text{ and } M=\begin{bmatrix} M_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

Now the Eq (2) can be expressed as;

$$EI\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}-w^2\begin{bmatrix} M_{11} & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}=0$$

Solving above equation

$$EI([K_{11}]\{U_1\}+[K_{12}]\{U_2\})-w^2[M_{11}]\{U_1\}=0 \qquad \text{Eq. (3)}$$

$$EI([K_{21}]\{U_1\}+[K_{22}]\{U_2\})-w^2(0)=0$$

$$\{U_2\}=-[K_{22}]^{-1}[K_{21}]\{U_1\}$$

Equation (3) becomes

$$EI([K_{11}]\{U_1\}+[K_{12}](-[K_{22}]^{-1}[K_{21}]\{U_1\}))-w^2[M_{11}]\{U_1\}=0$$

If $[\overline{K_{11}}]=[K_{11}]-[K_{12}][K_{22}]^{-1}[K_{21}]$

$$EI[\overline{K_{11}}]\{U_1\}=w^2[M_{11}]\{U_1\} \qquad \text{Eq. (4)}$$

The Eq. (4) represents the solution of general Eigen values. The matrix $[M_{11}]$ is symmetric and positive, the matrix $[\overline{K_{11}}]$ is real symmetric therefore, Cholesky method is adopted for analysis. Decompose $[M_{11}]$ into two triangular matrices, i.e. $[M_{11}]=[L][L]^T$, Where $[L]$ is an upper triangular matrix. Since $[M_{11}]$ is a diagonal matrix.

$$[L]=\begin{bmatrix} M_{11}^{1/2} & & & & 0 \\ & M_{22}^{1/2} & & & \\ & & , & & \\ & & & , & \\ 0 & & & & M_{nn}^{1/2} \end{bmatrix}=[M_{11}]^{1/2}$$

Then assuming that $\{U_1\}=[M_{11}]^{-1/2}\{Z\}$

Putting value of $\{U_1\}$ in Eq. (4)

$$EI[\overline{K_{11}}][M_{11}]^{-1/2}\{Z\}=w^2[M_{11}][M_{11}]^{-1/2}\{Z\}$$

$$EI[\overline{K_{11}}][M_{11}]^{-1/2}\{Z\}[M_{11}]^{-1/2}=w^2\{Z\}$$

$$A = EI\left[\overline{K_{11}}\right]\left[M_{11}\right]^{-1/2}\left[M_{11}\right]^{-1/2}$$

$$A\{Z\} = W^2\{Z\} \qquad \text{Eq. (5)}$$

Here, the matrix $[A]$ is a real symmetric matrix and the Eq. (5) is a solution of eigenvalues and eigenvectors. Finally, the vibration modes are normalized, i.e. $\{U_i\}^T[M_{11}]\{U_i\}=1$. A Schematic of the complete solution methodology is given in Figure 4.

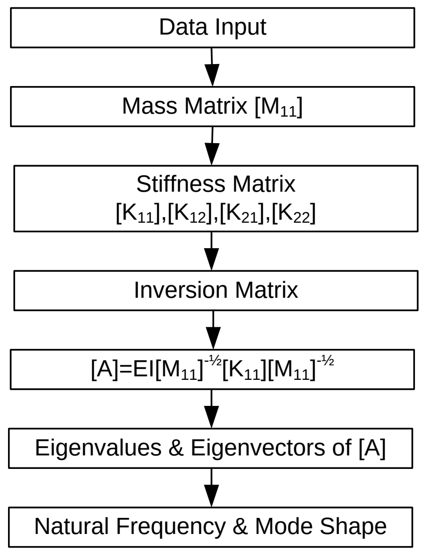

Figure 4: Model Flow Diagram

Stiffness Matrix

The fuel rod is discretized with beam element. Since the distance between the neighboring springs is only 1~2 times larger than the outside diameter of the fuel rod, therefore shear deformation of the beam is considered while calculating deflections and rotations of the beam. For this reason short beam elements (line elements) are used when using the FE method to calculate the fuel rod natural frequencies. It has six degrees of freedom (DoF) at each node, which includes translations and rotations along the x, y, and z directions, respectively. The stiffness matrix of the short beam element can be expressed as;

$$[K]_{l_i}^{e} = \frac{EI}{(1+b_i)\, l_i}\begin{bmatrix} 12 & 6l_i & -12 & 6l_i \\ 6l_i & (4+b_i)l_i^2 & -6l_i & (2-b_i)l_i^2 \\ -12 & -6l_i & 12 & -6l_i \\ 6l_i & (2-b_i)l_i^2 & -6l_i & (4+b_i)l_i^2 \end{bmatrix} \qquad \text{Eq. (6)}$$

Where

$[K]_{li}^{e}$ Stiffness matrix of the beam element,

l_i. Span length of beam element

E Young's modulus of elasticity for fuel rod

$bi = \frac{12kEI}{GAl_i}$

I Inertia moment of the beam cross section

G Shear elastic modulus of the fuel rod

A Cross section area of the beam element *k* Co-efficient due to non-uniform distributed shear stress on the beam cross section

The stiffness matrix represents a beam element, which is under pure bending load (no axial or torsion loads). Nodal displacements and nodal moment are related as:

$$\begin{bmatrix} V_i \\ M_i \\ V_{i+1} \\ M_{i+1} \end{bmatrix} = [K]^e_{l_i} \begin{bmatrix} v_i \\ \Phi_i \\ v_{i+1} \\ \Phi_{i+1} \end{bmatrix}$$

Where Vi , v_i are the nodal displacements of the *i*-the node, M_{i+1}, Φ_i are respectively the nodal moment and rotation of the *i*-th node etc. generalized equation for the continuous beam is as follows;

$$\{P_{1,} \quad P_{2,} \quad .. \quad P_{n,} \quad M_{1,} \quad M_{2,} \quad ... \quad M_{n,}\}^T =$$

$$[K]\{V_{1,} \quad V_{2,} \quad ..... \quad V_{n,} \quad \Phi_{1,} \quad \Phi_{2,} \quad ...... \quad \Phi_n\}^T$$

Where, the matrix $[K]$ is a global stiffness matrix and "P" and "M"are external nodal force and nodal moment. Expressions for all elements of the Global stiffness matrix are obtained after collecting and arranging all the nodal equilibrium equations.

Mass Matrix

A lumped mass technique is adopted which is a simpler method of approximation to replace the distributed inertia of the continuous system by a finite number of lumped inertia elements. All inertia effects are concentrated at the nodes. The nodes are assumed to be connected by elastic but mass less elements. Here rotational inertia terms in the mass matrix are omitted and hence the mass matrix is reduced to the form;

$$M = \begin{bmatrix} M_{11} & & & 0 \\ & M_{22} & & \\ & & , & \\ & & & M_{nn} \\ 0 & & & 0 \end{bmatrix}$$

Where; $M_{11} = \frac{1}{2} m_1 l_1, \quad M_{nn} = \frac{1}{2} m_{n-1} l_{n-1}$

$M_{ii} = \frac{1}{2}(m_{i-1} l_{i-1} + m_i l_i)$ and $(i = 2,3,...n-1)$

As described in the previous section that fuel stack will not contribute in bending moment because fuel stack is not a single intact part, it is divided into small fuel pallets which will not resist the bending of fuel rod. Therefore, bending moment and modulus of elasticity of fuel cladding is only considered for modal analysis. Input variable for calculation of modal analysis are given in Table 1.

Table1. Input Variable

Variable Name	Value
Elastic module of the fuel rod cladding material	1.02×10^{11} N/m^2
Shearing modulus of the fuel rod cladding material	3.57×10^{10} N/m^2
Moment of inertia of the fuel rod cladding cross section	2.22×10^{-10} m^4
Outer diameter of the fuel rod	0.01 m

Inner diameter of the fuel rod	0.0086 m
SG dimple stiffness	490 kN/m
SG spring stiffness	32.2 kN/m

The normal deflection curve, which determines the stiffness ratio (listed in Table 1) of the SG spring and dimple are shown in Figure 5. The results reveals that stiffness ratio of the dimple is much higher than the spring. It is also evident that dimple deflection curve is linear while spring deflection curve is non linear. This nonlinearity cannot be incorporated in free modal analysis.

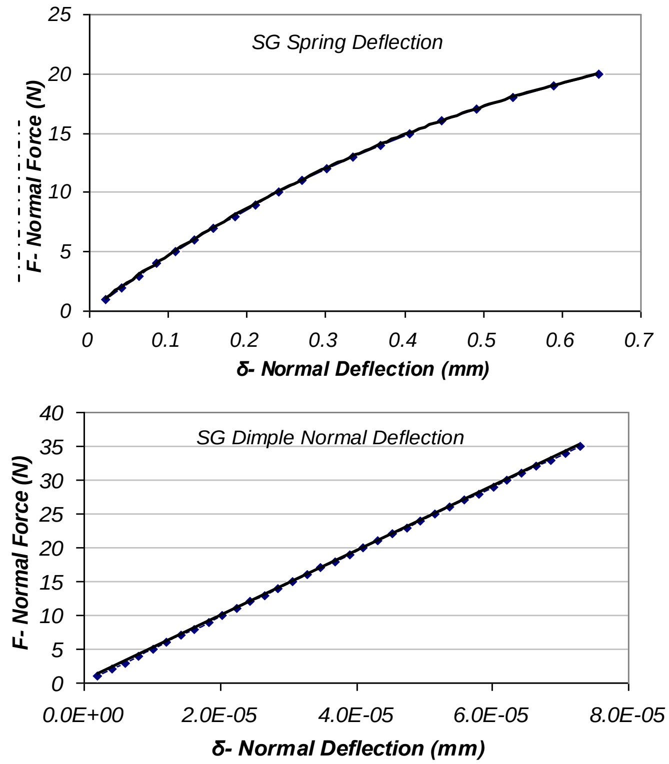

Figure 5: SG Spring & Dimple Deflection Curves

Therefore, the stiffness ratio of spring at 0.4 mm deflection is taken for analysis, which is initial deflection of the spring when fuel rod is inserted in the SG of fuel assembly.

Eigen Values and Eigen Vectors

Eigen values (Natural frequencies) and eigenvectors (mode shapes) of the fuel rod are calculated through solving the governing equations. Material and mechanical properties of the fuel rod and SG are attributed according to values listed in Table 1. The first six natural frequencies are listed in Table 2.

Table2. Natural Frequencies Fuel Rod using MATLAB

Mode	***Natural Frequency***
1st	*56.53*
2nd	*62.01*
3rd	*68.91*
4th	*77.98*
5th	*87.74*
6th	*95.97*

The normalized mode shapes with correspondent natural frequencies are given in Figure 6.

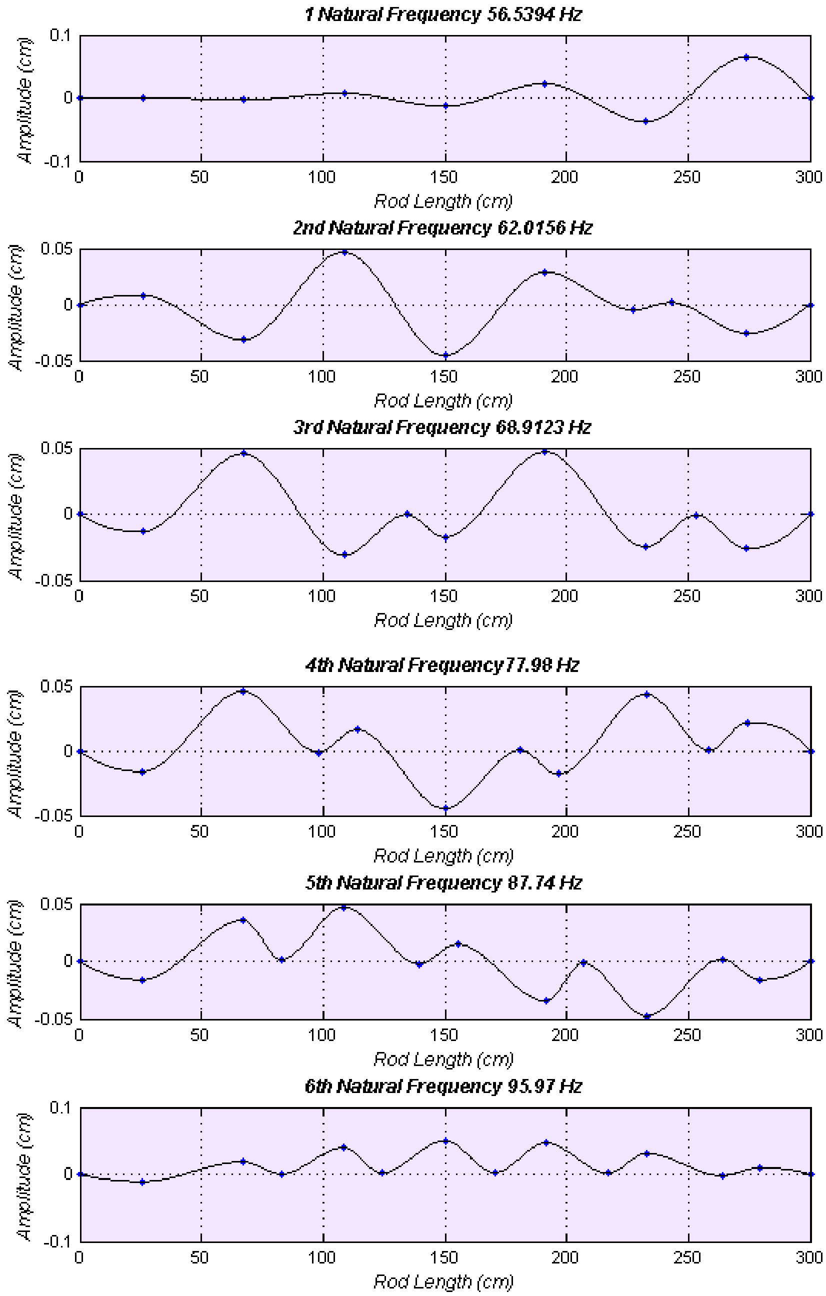

Figure 6: Mode Shape of CNPP Fuel Rod

3-D Modal analysis using ANSYS

3D modal analysis of CNPP fuel rod is also performed using ANSYS in order to find out the natural frequencies and mode shape. The methodology is consist of two steps first contact analysis is performed between the fuel rod and SG in ANSYS structural analysis module and then pre-stress vibration analysis is performed using the ANSYS modal analysis module.

Contact Analysis

Fuel rod is interferencely fitted between the SG spring and dimple. The vertical distance between the outer surface of the spring and opposite dimple is 9.6 mm and fuel rod outer diameter is 10mm. when the fuel rod is inserted between the SG it will deform 0.4mm in normal direction. In order to establish the frictional contact between the fuel rod and SG springs and dimples nonlinear contact analysis is performed.

In this regard 3-D solid modeling of CNPP SG cells and fuel rod is created in ANSYS design modeler as per actual geometry. It is described in the previous section that fuel section doesn’t contribute the bending moment therefore cladding of the fuel rod is modeled in ANSYS but mass of the fuel pellet is added in the cladding mass. All the material and mechanical properties are attributed in accordance with material and geometry of the SG and fuel rod, as listed in table 3. As a boundary condition the SG is constrained from right and left side (along the fuel rod axis) for all directional and rotational movements. Model is meshed with ‘Solid-186’ element and symmetric contact pair is created between fuel rod and SG at six points in SG cell (two at SG spring and four at SG dimple) with contact element ‘Conta-174’ and ‘Targe-170’. Contact arrangement of two SG straps (upper and lower) in a single cell is shown in Figure 7.

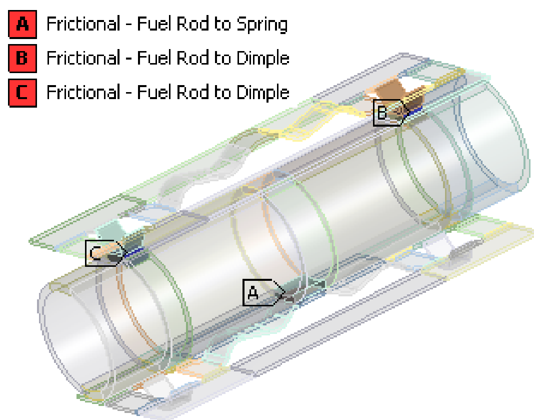

Figure 7: Contact Point in SG Cell

Same methodology is adopted for all eight SG cells along the fuel rod. A brief summary of FE model is given in given in Table 3.

Table3. ANSYS FE Model Summary

Element type for SG cell and fuel rod	Solid-186
Contact/Target element	Conta-174, Targe-170
Number of elements of ‘Solid-186’	15502
Number of elements of ‘Conta-174’	1073
Number of elements of solid ‘Targe-170’	455
Contact type	Frictional
Contact formulation	Augmented Lagrange
Modulus of elasticity of SG spring	2.0×10^5 MPa
Elastic modulus of the fuel rod cladding	1.02×10^5 MPa
Shearing modulus of the fuel rod cladding	3.567×104 MPa
Coefficient of friction	0.3

After contact analysis is performed, it is found that the spring is deformed only 0.38 mm (~0.4 mm) in normal direction, whereas the dimple deflection is almost negligible. The reason is that the normal stiffness of the SG dimple is much higher than the normal stiffness of SG dimple. Therefore, SG dimple is showing the rigid body motion behavior.

Figure 8 shows the front sectional view of the meshed fuel rod in contact with the SG spring. Before contact analysis spring is 0.4 mm inside the fuel rod but after contact analysis spring is deformed and adjusted to touch condition with fuel rod outer surface. Same contact creation is achieved for all contact pairs along the fuel rod.

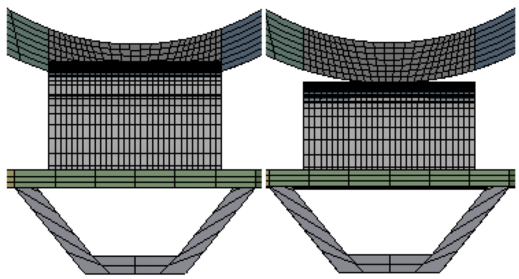

Figure 8: SG Spring Deformation

Pre-stressed Modal Analysis

Modal Analysis of fuel rod is performed after creating the contact between fuel rod SG. As a boundary condition SG axial edges along the fuel rod are constrained from all movements in all direction. Fuel rod is constrained to move in axial direction. Natural frequencies and mode shapes are extracted using Lanczos method.

Natural Frequencies and Mode Shapes

The first six mode shapes of CNPP fuel rod with corresponding calculated natural frequencies are shown in Figure 9.

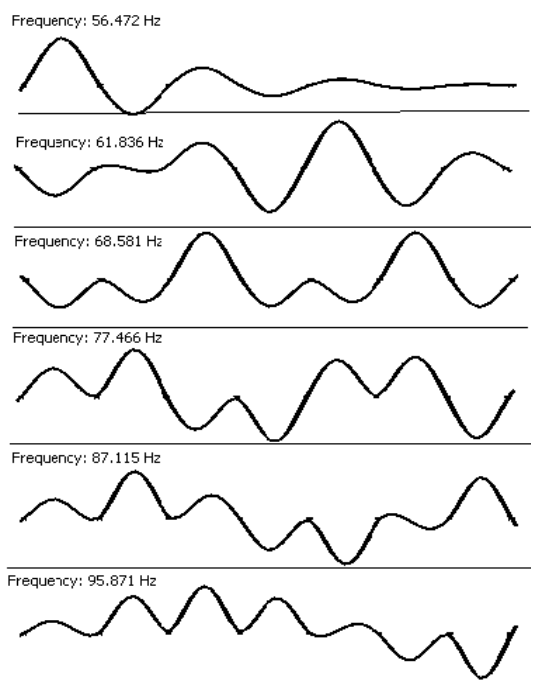

Figure 9: Fuel Rod Mode Shape using ANSYS

Comparison with Test Results

The mode shapes and natural frequencies of CNPP fuel rod calculated through MATLAB and ANSYS FE analyses have been compared with the test results conducted by Chinese A18.21 – 1994 "A Computer Code for the Calculation of Natural Frequencies and Modes of Fuel Rod Vibration Specifications". The comparison of results is listed in Table5 which depicts that analytical FE modal results are in good agreement with test results. Mode shapes obtained from both the analyses are also shows same pattern.

Table5. Natural Frequencies comparison

Mode	**Natural Frequencies (Hz)**		
	MATLAB	ANSYS	CHINESE TEST
1st	56.53	56.5	57.9
2nd	62.01	61.8	63.7
3rd	68.91	68.6	67.7
4th	77.98	77.5	78.3
5th	87.74	87.1	92.3
6th	95.97	95.9	99.8

Conclusions

Results obtained from the 2-D modal analysis accords with the 3-D ANSYS analysis and their concurrence with the Chinese test result enhanced the confidence in the FE analysis.

1) Fuel rod fundamental natural frequency is 56 Hz, which is higher than the expected excitation frequencies in the reactor (i.e., 0 to 50Hz), Kenard M. W et al, (1995).
2) Number of SG per fuel assembly and its stiffness has a significant effect on the modal parameters of fuel rod. The same conclusion was drawn by Kang H.S. et al (2000).
3) Increasing the number of SGs in a fuel assembly will decrease vibration amplitude of fuel rod because unsupported length of fuel rod will decrease and make fuel rod boundary conditions more stiffer. However, this will affect thermal hydraulic parameters and SG stiffness is the only parameter which controls the design of SG.

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